Microscopic description of the fission process:

the Schrödinger Collective Intrinsic Model (SCIM)

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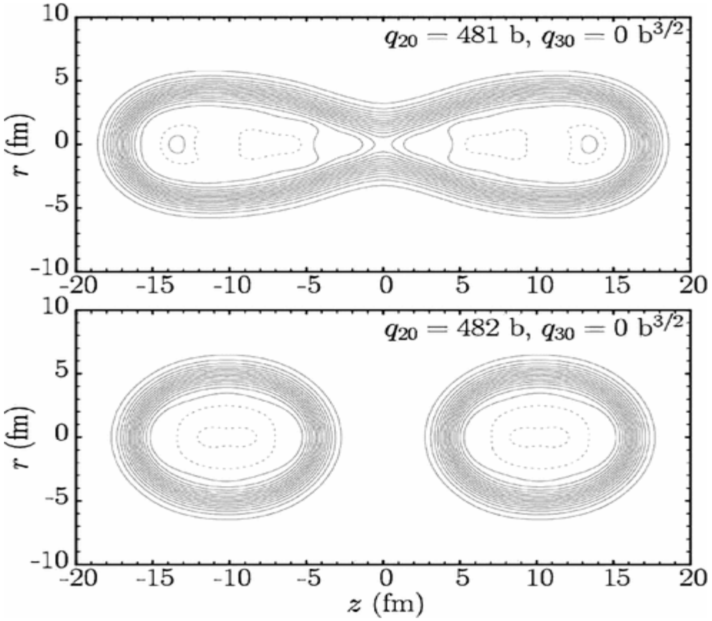
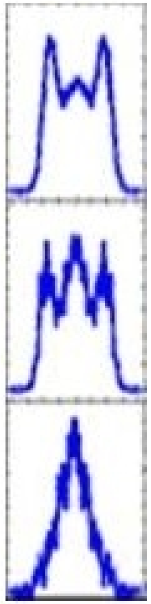
The Fission process

^{226}Th



N. Dubray et al, Phys.Rev. C 77, 014310 (2008)

N=136



Z=91

Z=90

Z=89

Charge distribution

K.H. Schmidt et al, Nucl. Phys. A665, 221 (2000)

Large amplitude motion

Results very sensitive to the number of nucleons

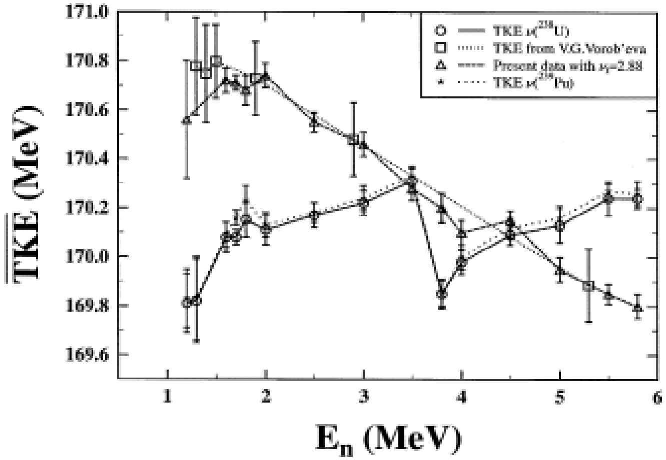
Collective variables needed

Microscopic approach needed

Intrinsic excitations during the fission process

* Experimental observables

Total Kinetic Energy drops suddenly



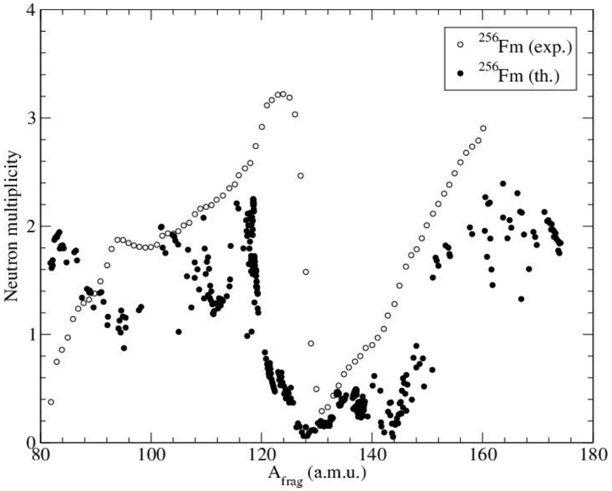
F. Vives, et al., Nucl Phys. A662 (2000) 63

S. Pomme et al., Nucl. Phys A560 (1993) 689

* Prompt neutrons emission

$$\nu = \frac{E_{def}}{E_k + B_n}$$

→ E_{int} is missing



N. Dubray et al., Phys. Rev. C 77 (2008) 014310

THE SCHRODINGER COLLECTIVE INTRINSIC MODEL

Generalization of the Generator Coordinate Method (GCM) to the intrinsic excitations

At each step of the collective motion the system is developed on a basis which includes the excited states:

$$|\Psi\rangle = \int dq\, f_0(q)|\varphi_0(q)\rangle + \sum_{i\neq 0} \int dq\, f_i(q)|\varphi_i(q)\rangle$$

↑ HFB minima

↑ HFB excited states

N. Tajima et al, Nucl. Phys. A, 542, 355 / H. Muther et al, Phys. Rev. C , 15, 1467

$f_i(q)$: Unknown functions

q: set of collective variables

⟶ Goal : Description of the fission

- **microscopic**
- **(time dependent)**
- **quantum mechanical**
- **non adiabatic**

Towards the Hill Wheeler equation

Introduction of the collective derivative operator :

$$P = i\hbar \frac{\partial}{\partial \bar{q}}$$

which acts only on the wave functions f

Taylor expansion $f(q') = e^{i\hbar s P/2} f(\bar{q})$ with $s = q - q'$, $\bar{q} = (q + q')/2$

The variational principle $\frac{\partial}{\partial f_i(\bar{q})} \frac{\langle \Psi | \hat{H} - E | \Psi \rangle}{\langle \Psi | \Psi \rangle} = 0$ leads to the Hill Wheeler equation :

$$\int ds\ e^{isP/2} [H(\bar{q} + s/2, \bar{q} - s/2) - EN(\bar{q} + s/2, \bar{q} - s/2)] e^{isP/2} f(\bar{q}) = 0$$

- Non local integral equation
- Matricial equation

with $H_{i,j}(q,q') = \langle \varphi_i(q) | \hat{H} | \varphi_j(q') \rangle$ Hamiltonian (kernel) matrix

$N_{i,j}(q,q') = \langle \phi_i(q) | \phi_j(q') \rangle$ Overlap (kernel) matrix

The Hill Wheeler equation

The Hamiltonian and overlap kernels H(q), N(q) are expressed with the help of their moments and of the Symmetric Ordered Products of Operators

$$A^{(n)}(\bar{q})=i^n\int_{-\infty}^{+\infty} ds\ s^n A(\bar{q}+s/2,\bar{q}-s/2) \qquad \left[A^{(n)}(\bar{q})P\right]^{[n]}=\frac{1}{2^n}\sum_q C_n^q P^{n-q}A^{(n)}(\bar{q})P^q$$

with such notation, the Hill Wheeler equation writes :

$$\boxed{(\underbrace{\sum_n \frac{1}{n!}\left[H^{(n)}(\bar{q})P\right]^{[n]}}_{\hat{H}(\bar{q})}-E\underbrace{\sum_n \frac{1}{n!}\left[N^{(n)}(\bar{q})P\right]^{[n]}}_{\hat{N}(\bar{q})})f(\bar{q})=0}$$

- H(q), N(q) hermitian
- Exact equation
- Matricial
- Odd and even moments included

in the present work

Reduction to a Schrödinger like equation

Defining the normalized wave function g such as $g(q) = \hat{N}^{1/2}(q) f(q)$ the Schrödinger equation is derived from the Hill Wheeler equation :

$$[\hat{J}^{+}_{-1/2}(\bar{q}) \frac{1}{\sqrt{N^{(0)}(\bar{q})}} \left(H^{(0)}(\bar{q}) + [H^{(1)}(\bar{q}) P]^{(1)} + \frac{1}{2} [H^{(2)}(\bar{q}) P]^{(2)} \right) \frac{1}{\sqrt{N^{(0)}(\bar{q})}} \hat{J}_{-1/2}(\bar{q}) - E] g(\bar{q}) = 0,$$

with the condition $\hat{J}^{+}_{-1/2}(\bar{q}) \hat{J}(\bar{q}) \hat{J}_{-1/2}(\bar{q}) = I$

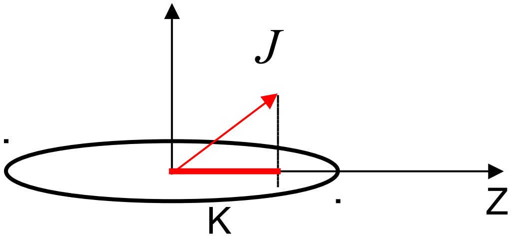
where a truncation of the Hamiltonian and overlap kernels is performed up to the symmetric product of "order 2 "

with $$\hat{J}(\bar{q}) = I + \frac{1}{\sqrt{N^{(0)}(\bar{q})}} \left([N^{(1)}(\bar{q}) P]^{[1]} + \frac{1}{2} [N^{(2)}(\bar{q}) P]^{[2]} \right) \frac{1}{\sqrt{N^{(0)}(\bar{q})}}$$

Determination of the excited HFB states $|\Phi_i(q)\rangle$ (in even-even nuclei)

$$|\varphi_i(q)\rangle = \alpha_i \left(\eta^+_{i_1}(q)\,\eta^+_{\bar{i}_2}(q) + \eta^+_{i_2}(q)\,\eta^+_{\bar{i}_1}(q) \right) |\varphi_0(q)\rangle$$

with i_1 a state and $\bar{i}_1$ its time reversed state



- Preserves the time reversal and axial symmetries

- 4, 6 … qp neglected, $E_i < 10\,MeV$

- Should have K=0 with K the projection of the angular momentum onto the symmetry axis ie $K_{i1}=K_{i2}$

- No self-consistent blocking: the average particle number of the excitation can differ from the mean value of the HFB ground state.

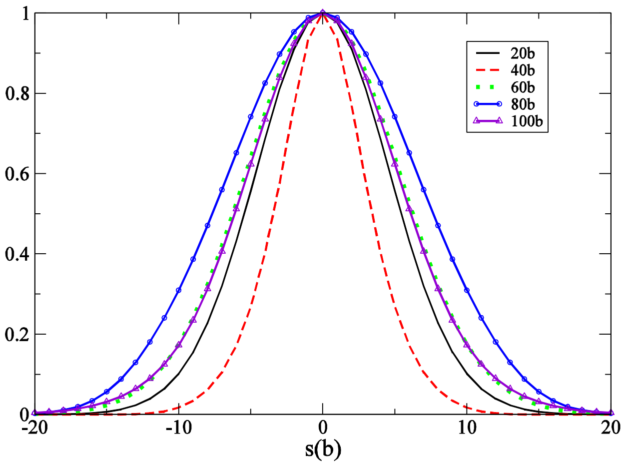
→ two kinds of excitations taken into account: pairing vibration type $\left(ie \quad i_2 = \bar{i}_1\right)$ and ph RPA type

Results on the overlap matrix in 236U

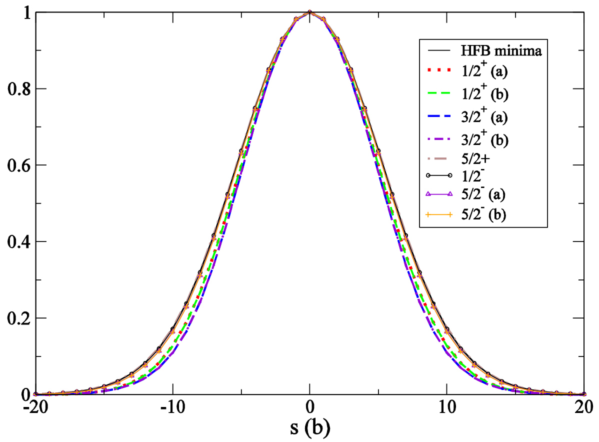
1) Diagonal terms $N_{i,i}(\bar{q}+s/2,\bar{q}-s/2)=\langle\phi_i(q)|\phi_i(q')\rangle$

$i=0: \quad \langle\phi(q)|\phi(q')\rangle$

$i\neq 0: \quad \langle\phi_i(q)|\phi_i(q')\rangle$



$20b \leq q \leq 100b$

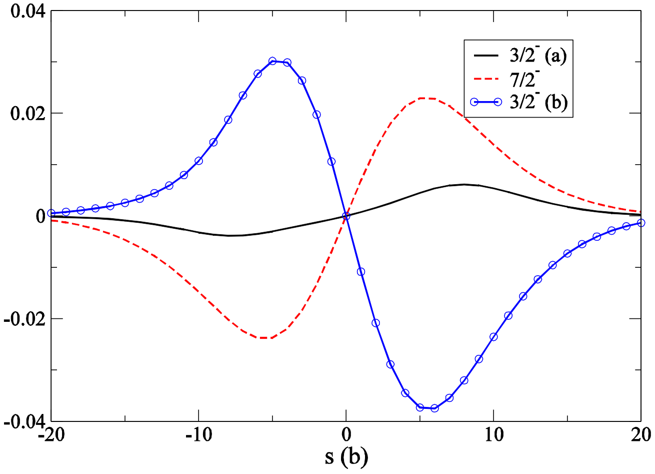


Overlaps between HFB minima and different 2qp excitations (labeled by K^{Π}(i)) at $q=60b$

- → Dependence in deformation
- → Diagonal overlaps not sensitive to excitations (except a few exceptions: level repulsions)

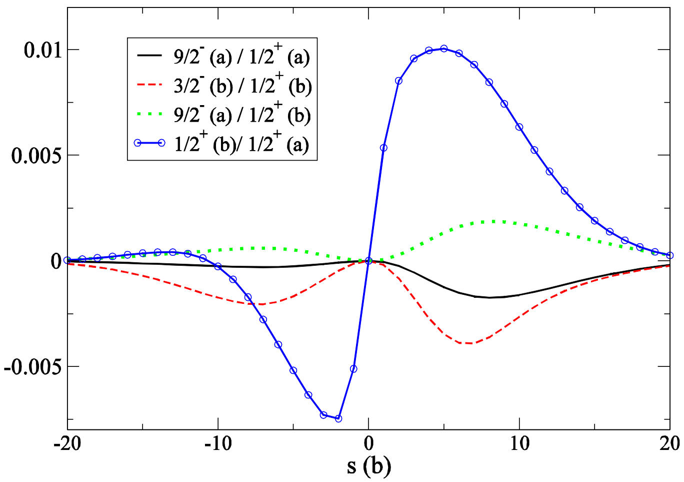
2) Non diagonal terms of the overlap matrix $N_{ij}(\bar{q}+s/2,\bar{q}-s/2)$

$\bar{q}=30b$



$N_{0,i}(\bar{q}+s/2,\bar{q}-s/2)$

$\bar{q}=30b$



$N_{i,j}(\bar{q}+s/2,\bar{q}-s/2)$

→ Strong dependence of the overlap in deformation and in excitations.

→ The moments derivatives can't be neglected.

→ Formalism key point:
Formal calculation of $\hat{J}_{-1/2}(\bar{q})$ at the 4th order in the development in moments

Bernard et al, PRC 84, 044308 (2011)

Determination of $\hat{J}_{-1/2}(\bar{q})$

$\hat{J}_{-1/2}$ must be solution of

$$\boxed{\hat{J}^{+}_{-1/2}(\bar{q})\hat{J}(\bar{q})\hat{J}_{-1/2}(\bar{q})\approx I \qquad (*)}$$

⟶ Set of non linear coupled equations

We set $$\hat{J}_{-1/2}(q)=\sum_{n=0}^{4}[j_n(q)P]^{(n)}$$

The solution of (*) is found by assuming:

$$\left(N^{(1)}\right)^{(p)}=0,\ for\ \ p\geq 1$$

$$\left(N^{(2)}\right)^{(p)}=0,\ for\ \ p\geq 2$$

Then the solution is restricted to the second order:

$$\boxed{\hat{J}_{-1/2}=I-\frac{1}{2}[WP]^{(1)}-[BP]^{(2)}}$$

with

$$W=F_1\left(N^{(0)},N^{(1)},N^{(2)}\right) \qquad B=F_2\left(W,N^{(0)\prime},N^{(2)\prime}\right)$$

Collective Intrinsic Hamiltonian of the Schrödinger equation

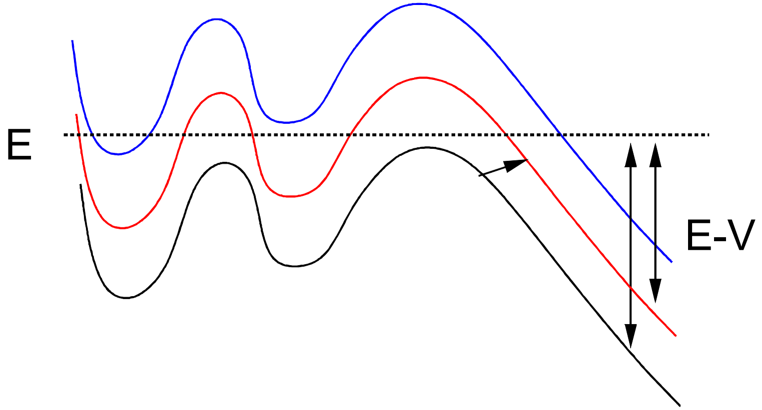
Diagonal terms: $H_{ii}^{CI}(q)=[(\frac{1}{2M(q)})_{ii} P]^{(2)} + V_{ii}(q), \quad (H^{CI}(q)-E)g(q)=0$

$P=i\hbar\frac{\partial}{\partial q}$ acts on $g(q)$

The coupling between the 2qp excited surfaces are defined considering the non diagonal elements, written as:

$$H_{ij}^{CI}(q)=[(\frac{1}{2M(q)})_{ij} P]^{(2)} + \underset{\downarrow}{[T_{ij}(q)P]^{(1)}} + V_{ij}(q)$$

Linear term in P



Properties of the Hamiltonian :

- Hermitian
- Time reversal invariant
- No dissipation; explicit treatment of the coupling between HFB g.s and 2qp excitations

Diagonal terms of the Collective Intrinsic Hamiltonian

Diagonal terms write:

$$H^{CI}_{ii}(\bar{q}) = \left[\left(\frac{1}{2\,M(\bar{q})} \right)_{ii} P \right]^{(2)} + V_{ii}(\bar{q})$$

Renormalization of the mass and the potential by the excitations

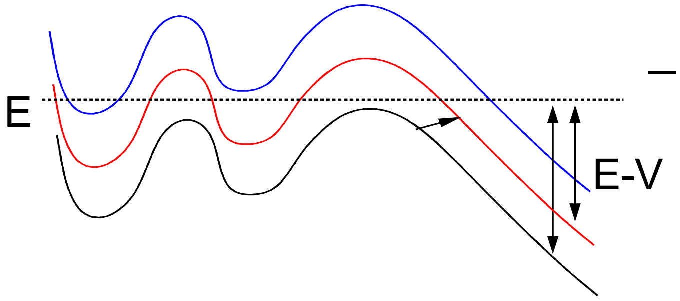
Scalar case

$$V^{sca}_{00} = h^{sca}{}_{(0)} + \frac{1}{4} h^{(2)\,sca}_{(0)} \, j_{(2)}{}^{sca} + \frac{1}{16} j^{(1)\,sca}_{(2)} \, h^{(2)\,sca}_{(0)} \, j^{(1)\,sca}_{(2)}$$

Matricial case

$$V^{CI}_{00} = (h_{(0)})_{00} + \frac{1}{4} (h^{(2)}_{(0)} j_{(2)})_{00} + \frac{1}{16} (j^{(1)}_{(2)} h^{(2)}_{(0)} j^{(1)}_{(2)})_{00}$$

$$- \frac{1}{2} (h^{(1)}_{(0)} j_{(1)})_{00} - \frac{1}{16} (j_{(1)} h^{(2)}_{(0)} j_{(1)})_{00} + \frac{1}{16} (j_{(1)} h^{(2)}_{(0)} j^{(1)}_{(2)})_{00}$$



Conclusion and outlooks

Present

Derivation of a new formalism: the coupling between intrinsic excitations and collective modes is determined microscopically. The sole ingredient is the effective force.

Future

- Evaluation of $\langle \phi_i(q) | \hat{H} | \phi_j(q') \rangle$. Reduction of the collective intrinsic Hamiltonian
- Calculations of the new inertia
(+ comparison with the usual adiabatic case)

Further future

- 1D calculation along a fission barrier.
- Other applications of the SCIM: spectroscopy (low lying 0+2 states, decay of superdeformed states)