Continuum RPA with finite-range interactions

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Single particle configuration space. Hartree-Fock.

$$\mathcal{H}\left[\phi_k(\mathbf{r})\right] = -\frac{\hbar^2}{2m}\nabla^2\phi_k(\mathbf{r}) + \mathcal{U}(\mathbf{r})\phi_\mathbf{k}(\mathbf{r}) - \int \mathrm{d}^\mathbf{3}\mathbf{r}'\,\mathcal{W}(\mathbf{r},\mathbf{r}')\,\phi_\mathbf{k}(\mathbf{r}') = \epsilon_\mathbf{k}\,\phi_\mathbf{k}(\mathbf{r})\,,$$

$$\mathcal{U}(\mathbf{r}) = \sum_{\alpha=1}^{8}\sum_{j\leq F}\int \mathrm{d}^3 r'\,\phi_j^*(\mathbf{r}')\,V_\alpha(\mathbf{r},\mathbf{r}')\,\phi_j(\mathbf{r}')$$

$$\mathcal{W}(\mathbf{r},\mathbf{r}') = \sum_{\alpha=1}^{8}\sum_{j\leq F}\phi_j^*(\mathbf{r}')\,V_\alpha(\mathbf{r},\mathbf{r}')\,\phi_j(\mathbf{r})$$

$$V_\alpha(\mathbf{r}_i,\mathbf{r}_j) = v_\alpha(|\mathbf{r}_i-\mathbf{r}_j|)\,O^\alpha_{i,j}\,,\ \text{ with } \alpha = 1,2,\ldots,7$$

$$\begin{aligned} O^\alpha_{i,j} \quad : \quad & 1\,,\ \boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j)\,,\ \boldsymbol{\sigma}(i)\cdot\boldsymbol{\sigma}(j)\,,\ \boldsymbol{\sigma}(i)\cdot\boldsymbol{\sigma}(j)\,\boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j)\,,\\ & S(i,j)\,,\ S(i,j)\boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j)\,,\ \mathbf{l}_{ij}\cdot\mathbf{s}_{ij} \end{aligned}$$

$$\phi^t_k(\mathbf{r}) = \phi^t_{nljm}(\mathbf{r}) = R^t_{nlj}(r) \sum_{s\mu} \langle l\mu \frac{1}{2} s | jm \rangle \, Y_{l\mu}(\Omega) \, \chi_s$$

k indicates all the quantum numbers but the s.p. energy

Closure relation

$$\sum_{\epsilon_i < \epsilon_\mathsf{F}} \delta_{ik} \, R_i(r) \, R^*_i(r') + \sum\!\!\!\!\!\!\int_{\epsilon_k} R_k(r, \epsilon_k) \, R^*_k(r', \epsilon_k) = \delta(r - r')$$

$$\sum\!\!\!\!\!\!\int_{\epsilon_k} \equiv \sum_{\epsilon_\mathsf{F} \le \epsilon_k \le 0} + \int_0^\infty \mathrm{d}\epsilon_k$$

Continuum RPA

$$Q_\nu^\dagger = \sum_{ph} \sum\!\!\!\!\!\!\int_{\epsilon_p} \left[X^\nu_{ph}(\epsilon_p)\, a_p^\dagger(\epsilon_p)\, a_h \; - \; Y^\nu_{ph}(\epsilon_p)\, a_h^\dagger\, a_p(\epsilon_p) \right] \, ,$$

$$|\nu\rangle \equiv |J, \Pi, \omega\rangle = Q_\nu^\dagger\, |0\rangle$$

Secular equations

$$(\epsilon_p - \epsilon_h - \omega)\, X^\nu_{ph}(\epsilon_p) + \sum_{p'h'} \sum\!\!\!\!\!\!\int_{\epsilon_{p'}} \left[v^J_{ph,p'h'}(\epsilon_p, \epsilon_{p'})\, X^\nu_{p'h'}(\epsilon_{p'}) \; + \; u^J_{ph,p'h'}(\epsilon_p, \epsilon_{p'})\, Y^\nu_{p'h'}(\epsilon_{p'}) \right] = 0$$

$$(\epsilon_p - \epsilon_h + \omega)\, Y^\nu_{ph}(\epsilon_p) + \sum_{p'h'} \sum\!\!\!\!\!\!\int_{\epsilon_{p'}} \left[v^{J*}_{ph,p'h'}(\epsilon_p, \epsilon_{p'})\, Y^\nu_{p'h'}(\epsilon_p) \; + \; u^{J*}_{ph,p'h'}(\epsilon_p, \epsilon_{p'})\, X^\nu_{p'h'}(\epsilon_{p'}) \right] = 0$$

New unknowns

$$f^\nu_{ph}(r) = \sum\!\!\!\!\!\!\int_{\epsilon_p} X^\nu_{ph}(\epsilon_p)\, R_p(r, \epsilon_p) \;\; , \;\; g^\nu_{ph}(r) = \sum\!\!\!\!\!\!\int_{\epsilon_p} Y^{\nu*}_{ph}(\epsilon_p)\, R_p(r, \epsilon_p) \, .$$

$$(\epsilon_p - \epsilon_h - \omega)\, X^{\nu}_{ph}(\epsilon_p) +$$

$$\sum_{p'h'} \sum\!\!\!\!\!\!\int_{\epsilon_{p'}} \left[v^J_{ph,p'h'}(\epsilon_p, \epsilon_{p'})\, X^{\nu}_{p'h'}(\epsilon_{p'}) \,+\, u^J_{ph,p'h'}(\epsilon_p, \epsilon_{p'})\, Y^{\nu}_{p'h'}(\epsilon_{p'}) \right] \,=\, 0$$

Multiply by $R_p(r, \epsilon_p)$

$$(\epsilon_p - \epsilon_h - \omega)\, R_p(r, \epsilon_p)\, X^{\nu}_{ph}(\epsilon_p) =$$

$$\mathcal{H} \left[R_p(r, \epsilon_p)\, X^{\nu}_{ph}(\epsilon_p) \right] \,-\, (\epsilon_h + \omega)\, R_p(r, \epsilon_p)\, X^{\nu}_{ph}(\epsilon_p)$$

Sum and integrate on ϵ_p.

$$\sum\!\!\!\!\!\!\int_{\epsilon_p} \mathcal{H} \left[R_p(r, \epsilon_p)\, X^{\nu}_{ph}(\epsilon_p) \right] = \mathcal{H} \left[f^{\nu}_{ph}(r) \right]$$

Apply these operations to all the terms of the secular equations

New secular equations

$$\mathcal{H}\left[f_{ph}(r)\right] - (\epsilon_h + \omega)\, f_{ph}(r) =$$
$$-\mathcal{F}^J_{ph}(r) + \sum_{\epsilon_i<\epsilon_\mathsf{F}} \delta_{ip}\, R_i(r) \int \mathrm{d}r_1\, r_1^2\, R_i^*(r_1)\, \mathcal{F}^J_{ph}(r_1)\,,$$

$$\mathcal{H}\left[g_{ph}(r)\right] - (\epsilon_h - \omega)\, g_{ph}(r) =$$
$$-\mathcal{G}^J_{ph}(r) + \sum_{\epsilon_i<\epsilon_\mathsf{F}} \delta_{ip}\, R_i(r) \int \mathrm{d}r_1\, r_1^2\, R_i^*(r_1)\, \mathcal{G}^J_{ph}(r_1)\,,$$

$$\mathcal{F}^J_{ph}(r) = \sum_{p'h'} \int \mathrm{d}r_2\, r_2^2$$
$$\left\{ R^*_{h'}(r_2) \left[V^{J,\mathrm{dir}}_{ph,p'h'}(r,r_2)\, R_h(r)\, f_{p'h'}(r_2) - V^{J,\mathrm{exc}}_{ph,p'h'}(r,r_2)\, f_{p'h'}(r)\, R_h(r_2) \right] \right.$$
$$\left. + g^*_{p'h'}(r_2) \left[U^{J,\mathrm{dir}}_{ph,p'h'}(r,r_2)\, R_h(r)\, R_{h'}(r_2) - U^{J,\mathrm{exc}}_{ph,p'h'}(r,r_2)\, R_{h'}(r)\, R_h(r_2) \right] \right\}$$

$\mathcal{G}^J_{ph}$ is obtained from the above equation by interchanging the f and g channel functions.

The original algebraic RPA secular equations depending on the continuous variable ϵ_p, have been written as integro-differential equations with unknowns depending on r.

Boundary conditions

$$f_{ph}^{p_0 h_0}(r \to \infty) \to R_{p_0}(r, \epsilon_{p_0} = \omega + \epsilon_{h_0})\, \delta_{p,p_0}\, \delta_{h,h_0} + \lambda\, H_p^-(\epsilon_h + \omega, r)$$

λ complex normalisation constant, $H_p^-(\epsilon_h + \omega, r)$ ingoing Coulomb function for protons, Hankel function for neutrons. $g_{ph}^{p_0 h_0}$,

$$g_{ph}^{p_0 h_0}(r \to \infty) \to \frac{1}{r} \exp\left[-r \left(\frac{2m|\epsilon_h - \omega|}{\hbar^2} \right)^{\frac{1}{2}} \right]$$

Sturm-Bessel functions

$$\left[-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{\hbar^2}{m}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} + \frac{\hbar^2}{2m}\frac{l_p(l_p+1)}{r^2} - \epsilon_p\right]\Phi_p^\mu(r) \;=\; -\overline{U}_p^\mu(r)\,\Phi_p^\mu(r)$$

$$\overline{U}_p^\mu(r) \;=\; \begin{cases} \beta_p^\mu + i\gamma_p^\mu\,, & \text{if } r \le a\,, \\ 0\,, & \text{if } r > a\,, \end{cases}$$

$$(\beta_p^\mu + i\gamma_p^\mu)\int_0^a \mathrm{d}r\, r^2\,\Phi_p^\mu(r)\,\Phi_p^\nu(r) \;=\; \delta_{\mu\nu}$$

$$\widetilde{\Phi}_p^\mu(r) \;=\; \Phi_p^\mu(r) - \sum_{\epsilon_i<\epsilon_\mathrm{F}} \delta_{ip}\,R_i^*(r)\int \mathrm{d}r'\,r'^2\,R_i(r')\,\Phi_p^\mu(r')$$

$$f_{ph}^{p_0h_0}(r) \;=\; R_{p_0}(r,\epsilon_{p_0})\,\delta_{pp_0}\,\delta_{hh_0} + \sum_\mu c_{ph}^{\mu+}\,\widetilde{\Phi}_p^{\mu+}(r)\,,$$

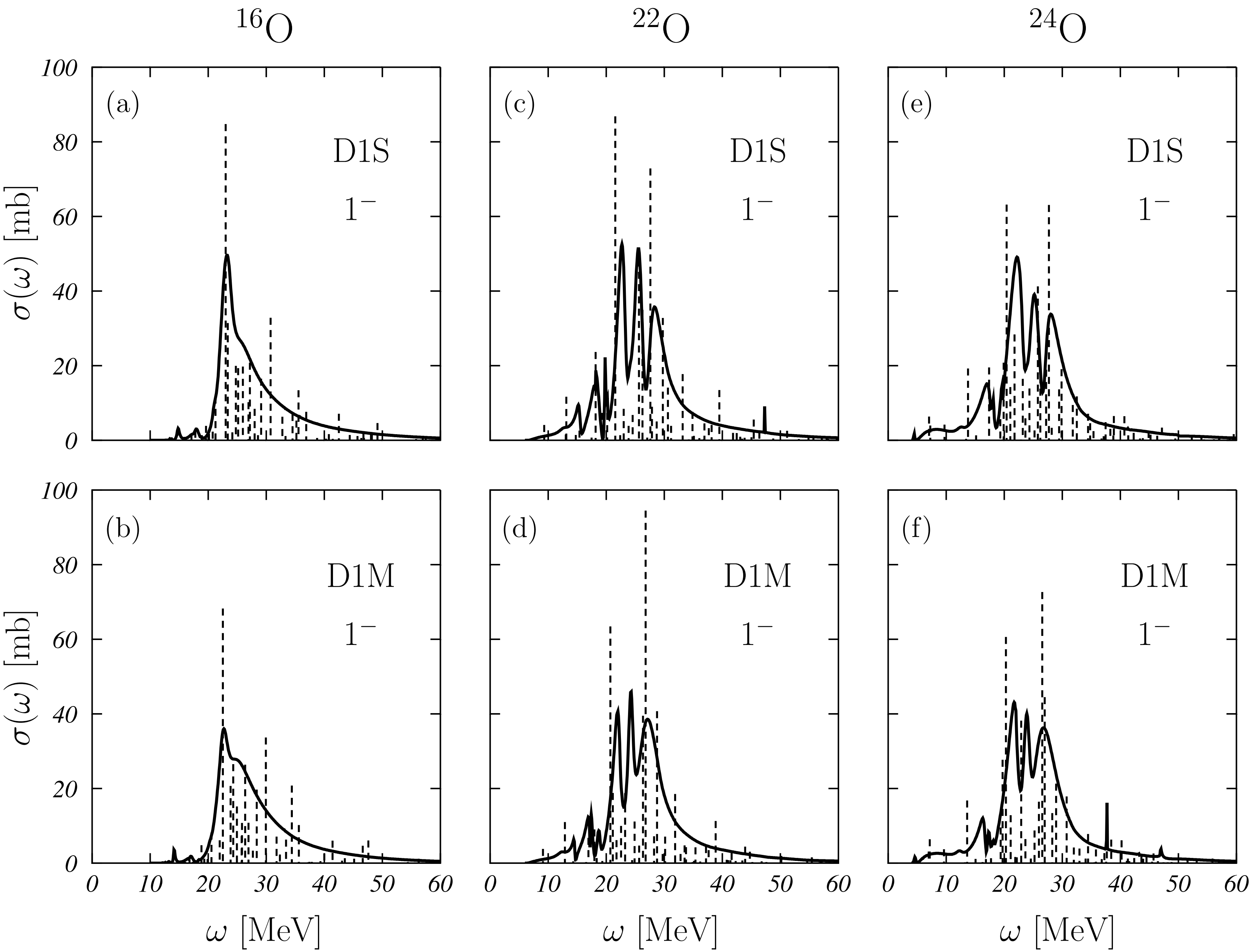
$$g_{ph}^{p_0h_0}(r) \;=\; \sum_\mu c_{ph}^{\mu-}\,\widetilde{\Phi}_p^{\mu-}(r)\,,$$

Observables

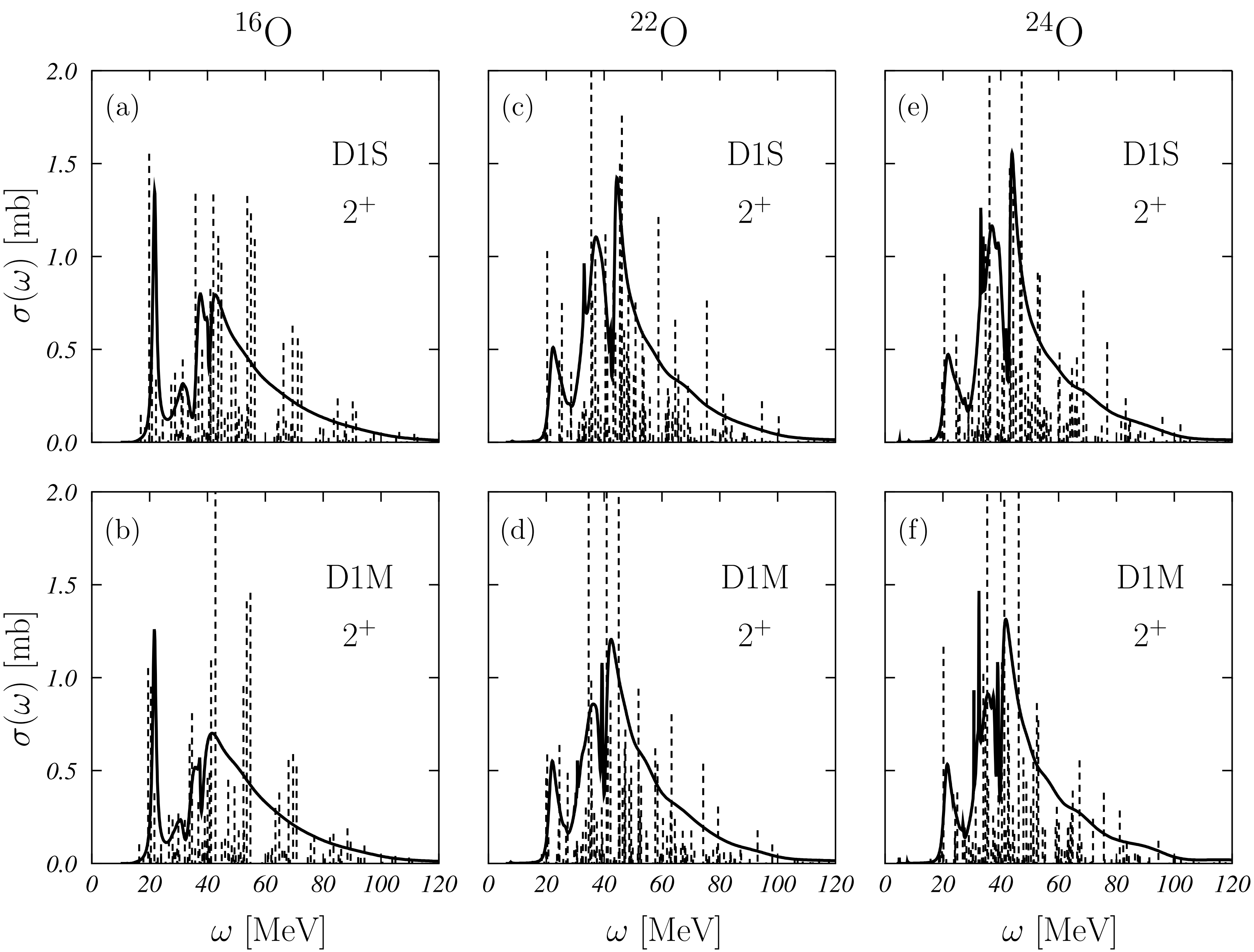
$$T_{JM}(\mathbf{r}) = \sum_{i=1}^{A} F_J(r_i)\, \theta_{JM}(\Omega_i)\, \delta(\mathbf{r}_i - \mathbf{r})$$

$$\begin{aligned}\langle J \| T_J \| 0 \rangle_{p_0 h_0} = \sum_{ph} \Big[& \langle j_p \| \theta_J \| j_h \rangle \int \mathrm{d}r\, r^2\, (f_{ph}^{p_0 h_0}(r))^*\, F_J(r)\, R_h(r) \\ & + (-1)^{J+j_p-j_h} \langle j_h \| \theta_J \| j_p \rangle \int \mathrm{d}r\, r^2\, R_h^*(r)\, F_J(r)\, g_{ph}^{p_0 h_0}(r) \Big]\end{aligned}$$

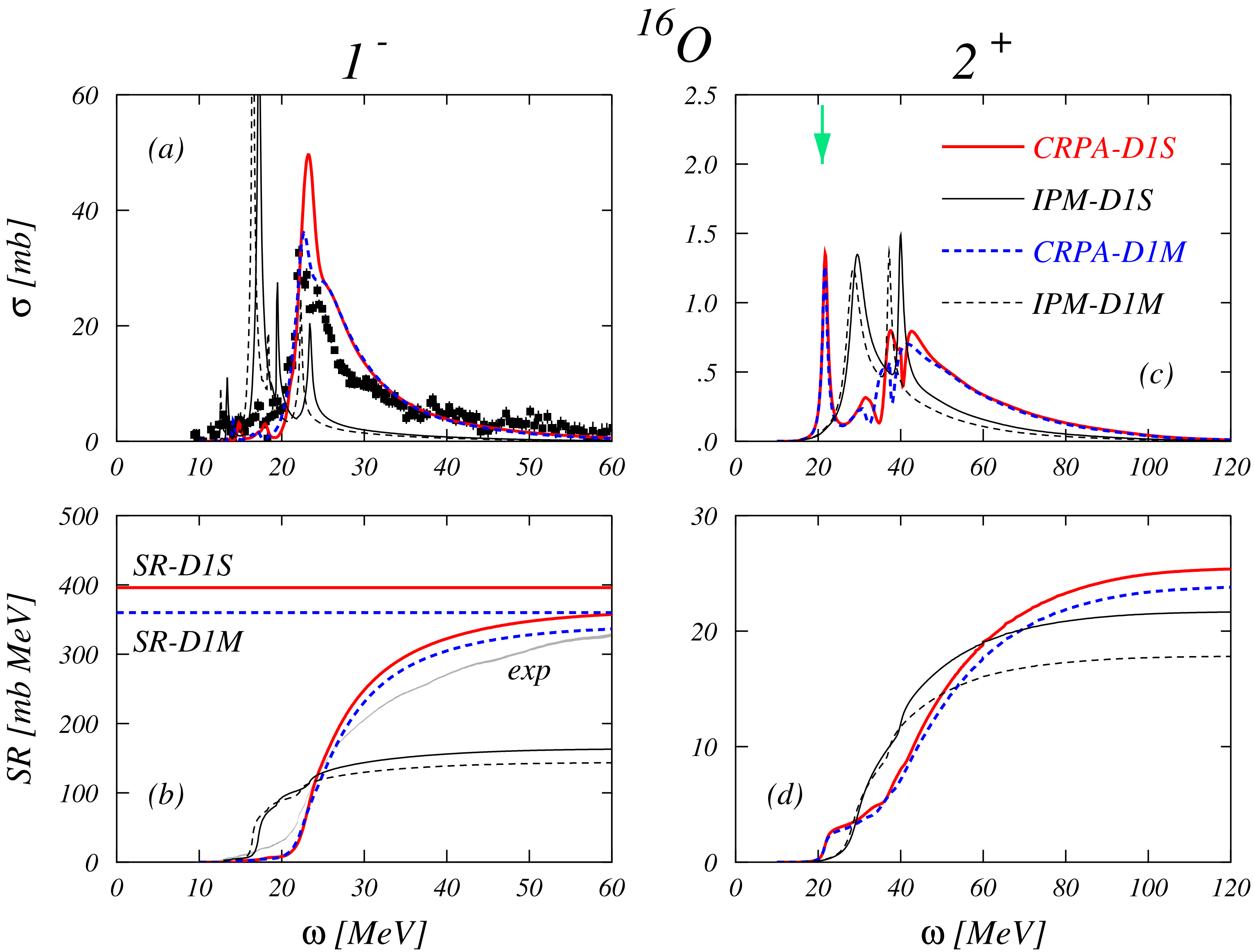
$$\sigma \sim \sum_{p_0 h_0} |\langle J \| T_J \| 0 \rangle_{p_0 h_0}|^2$$



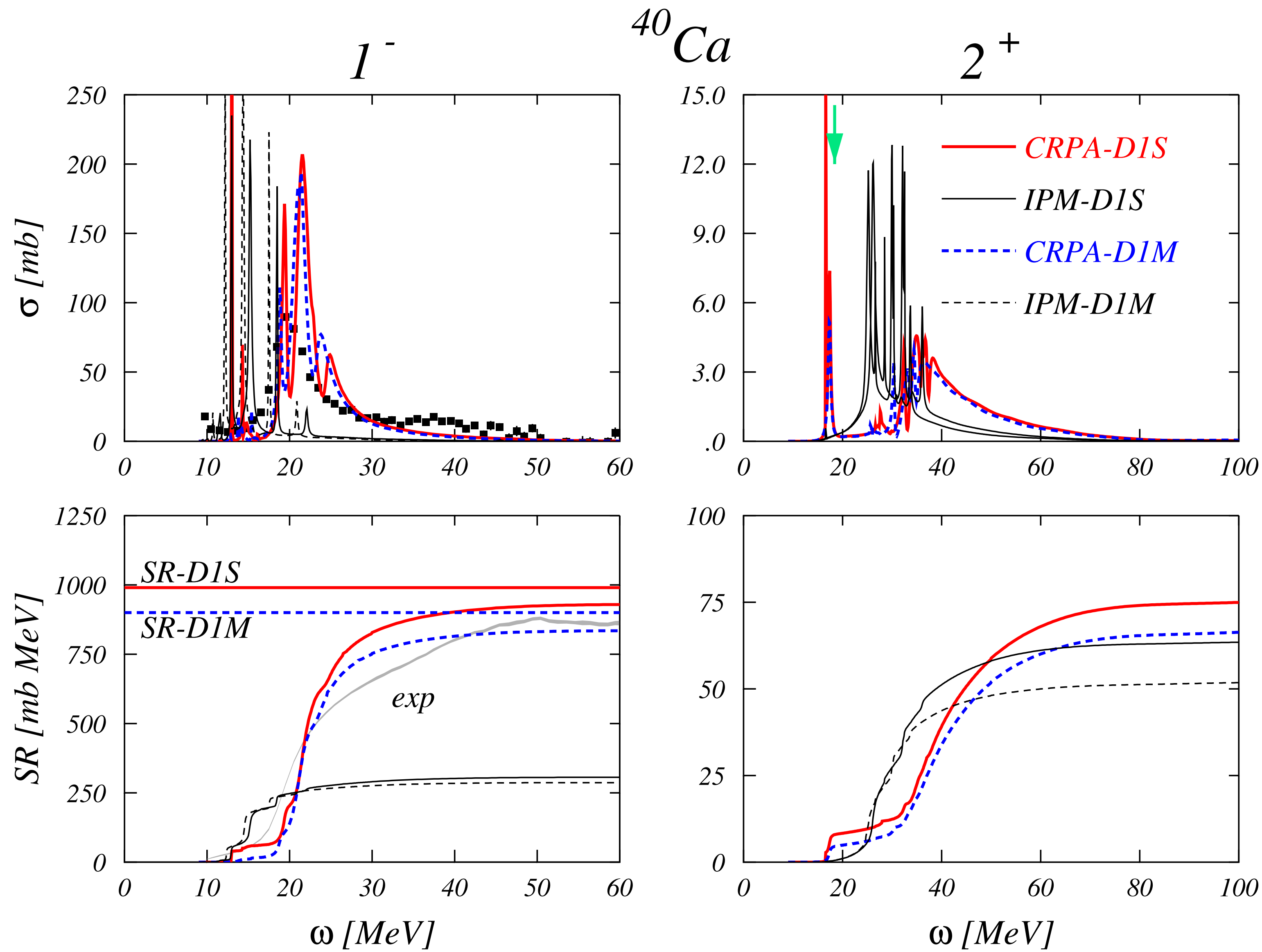
^{16}O
^{22}O
^{24}O
(a)
(b)
(c)
(d)
(e)
(f)
D1S
D1M
1^-
$\sigma(\omega)$ [mb]
ω [MeV]
0
20
40
60
80
100
0
10
20
30
40
50
60



^{16}O
^{22}O
^{24}O
(a)
(b)
(c)
(d)
(e)
(f)
D1S
D1M
2$^+$
$\sigma(\omega)$ [mb]
ω [MeV]
0.0
0.5
1.0
1.5
2.0
0
20
40
60
80
100
120



Data: J. Ahrens et al., Nucl. Phys. A 251 (1975) 479.



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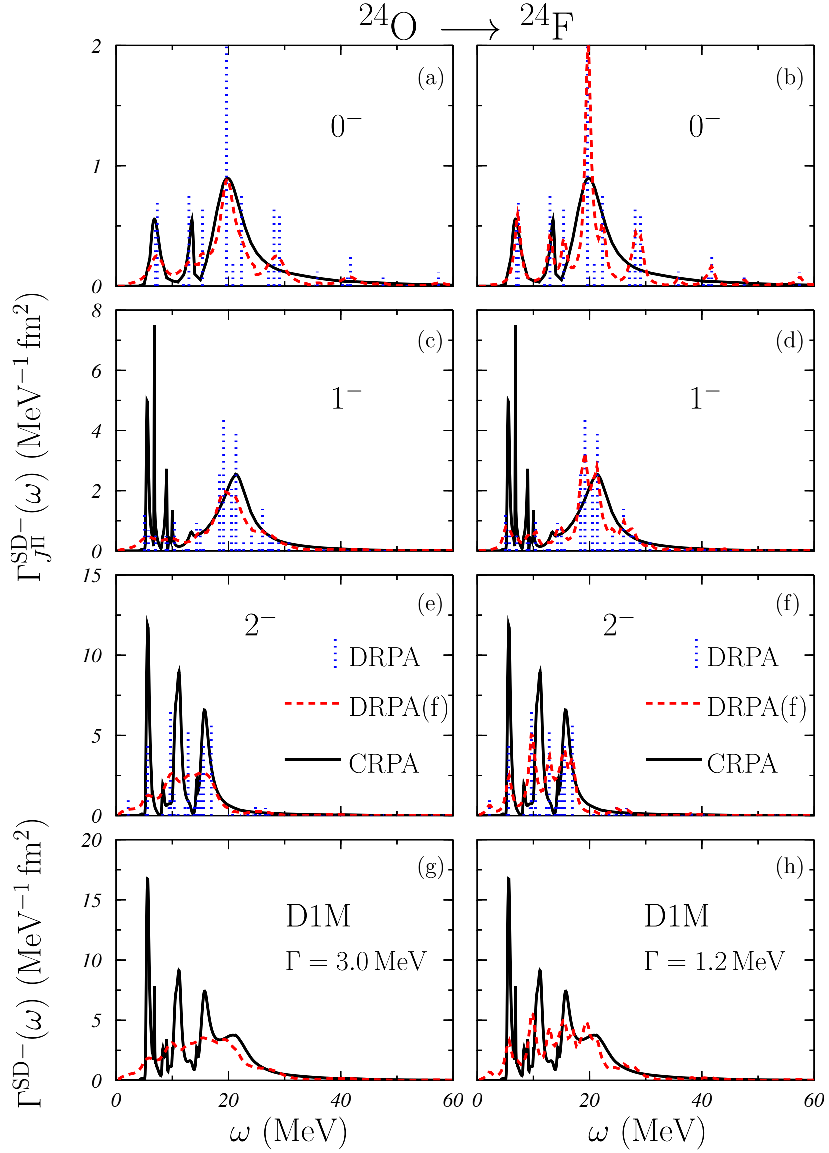
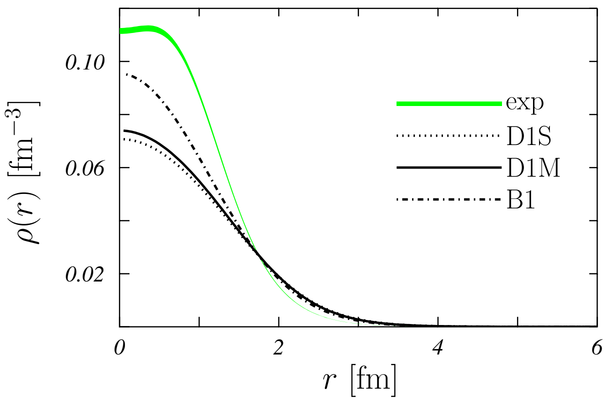
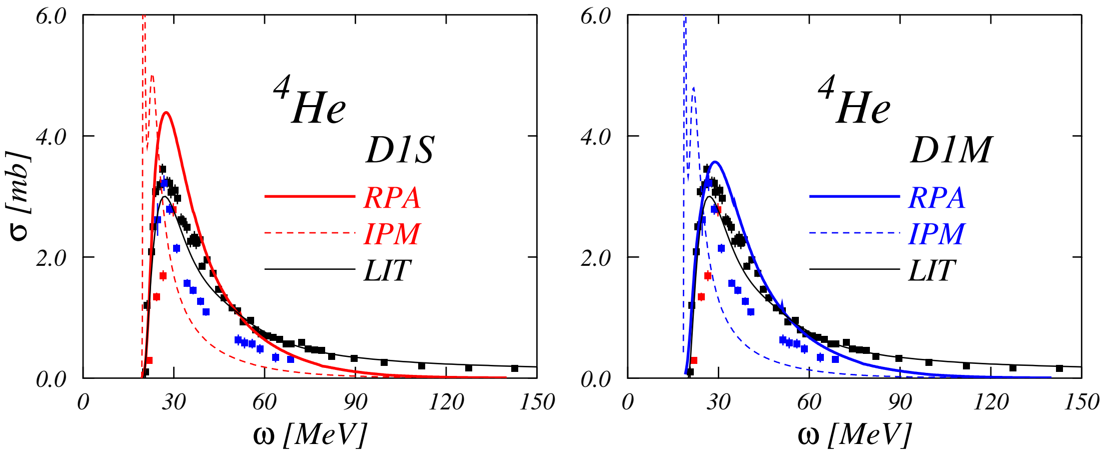


$^{24}O \longrightarrow {}^{24}F$
(a)
(b)
0^-
(c)
(d)
1^-
(e)
(f)
2^-
DRPA
DRPA(f)
CRPA
(g)
(h)
D1M
$\Gamma = 3.0$ MeV
$\Gamma = 1.2$ MeV
$\Gamma^{SD-}_{J\Pi}(\omega)$ (MeV^{-1} fm^2)
$\Gamma^{SD-}(\omega)$ (MeV^{-1} fm^2)
ω (MeV)



B/A (MeV)	
D1S	30.28
D1M	29.54
B1	28.48
exp	28.29



Photoabsorption data.

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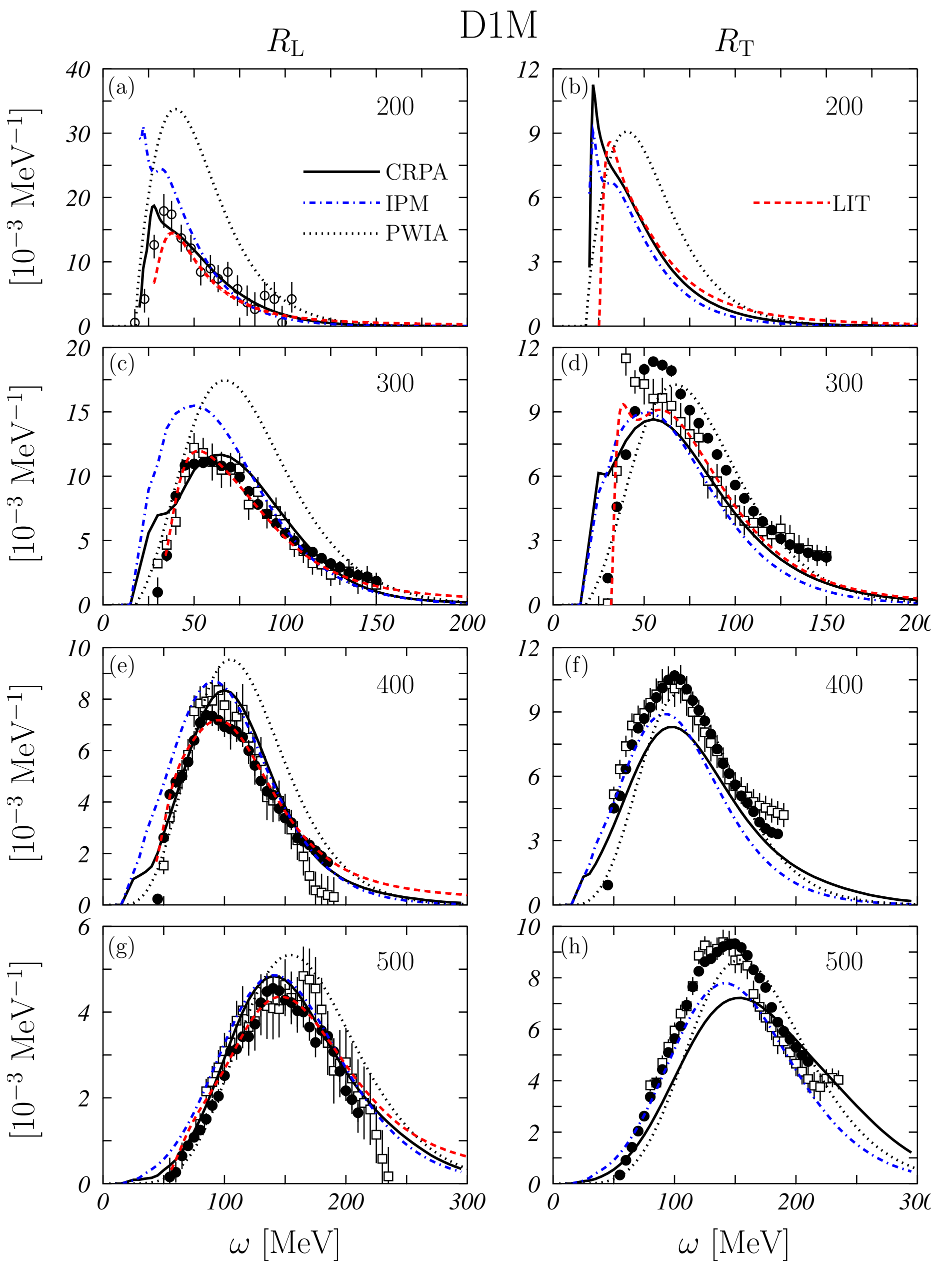

Data

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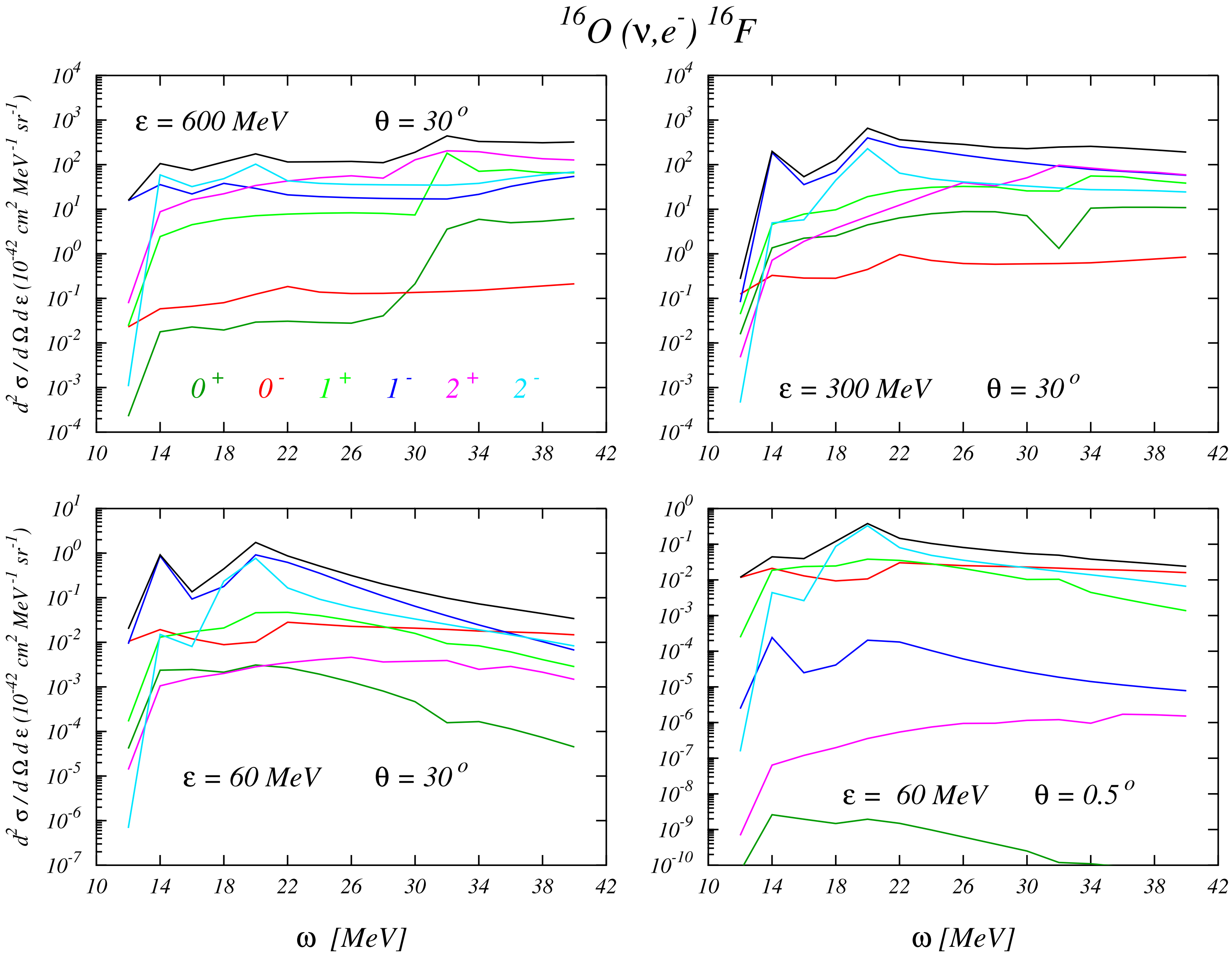

$^{16}O\,(\nu,e^-)\,^{16}F$
ε = 600 MeV θ = 30°
ε = 300 MeV θ = 30°
ε = 60 MeV θ = 30°
ε = 60 MeV θ = 0.5°
0+ 0- 1+ 1- 2+ 2-
d²σ / dΩ dε (10⁻⁴² cm² MeV⁻¹ sr⁻¹)
ω [MeV]