

$$\begin{aligned}
V_{12} = & \sum_{j=1}^2 e^{-(\vec{r}_1 - \vec{r}_2)^2 / \mu_j^2} (W_j + B_j P_\sigma - H_j P_\tau - M_j P_\sigma P_\tau) \\
& + t_3 (1 + x_0 P_\sigma) \delta(\vec{r}_1 - \vec{r}_2) \left[\rho \left(\frac{\vec{r}_1 + \vec{r}_2}{2} \right) \right]^\alpha \\
& + i W_{LS} \overleftarrow{\nabla}_{12} \delta(\vec{r}_1 - \vec{r}_2) \times \overrightarrow{\nabla}_{12} \cdot (\vec{\sigma}_1 + \vec{\sigma}_2) \\
& + (1 + 2\tau_{1z})(1 + 2\tau_{2z}) \frac{e^2}{|\vec{r}_1 - \vec{r}_2|}
\end{aligned}$$

First Gogny Conference,

Bruyeres le Châtel, Fr

Lights and shadows of the Gogny force



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The **Gogny force** is an effective, phenomenological density dependent interaction

$$V(\vec{r}_1 - \vec{r}_2) = V_C(1, 2) + V_{LS}(1, 2) + V_{Coul}(1, 2) + V_{DD}$$

$$V_C(\vec{r}_1 - \vec{r}_2) = \sum_i (W_i - H_i P_\tau + B_i P_\sigma - M_i P_\sigma P_\tau) \exp((\vec{r}_1 - \vec{r}_2)^2 / \mu_i^2)$$

$$V_{LS}(1, 2) = W_{LS}^i (\nabla_{12} \delta(\vec{r}_1 - \vec{r}_2) \nabla_{12}) (\vec{\sigma}_1 + \vec{\sigma}_2) \quad V_C(1, 2) = \frac{e^2}{4\pi\epsilon_0 r}$$

$$V_{DD}(1, 2) = t_3 \delta(\vec{r}_1 - \vec{r}_2) (1 + x_0 P_\sigma) \rho^\alpha(\vec{R})$$

Parameters fixed by imposing some nuclear matter properties and a few values from finite nuclei (binding energies, s.p.e. splittings and some radii information).

- **D1S:** surface energy fine tuned to reproduce fission barriers
- **D1N:** Realistic neutron matter equation of state reproduced
- **D1M:** Realistic neutron matter + Binding energies of essentially all nuclei with beyond mean field effects

Pairing and time-odd fields are taken from the interaction itself



The Gogny force: a successful mean field model

PHYSICAL REVIEW C **81**, 014303 (2010)

Structure of even-even nuclei using a mapped collective Hamiltonian and the DIS Gogny interaction

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PHYSICAL REVIEW LETTERS

week ending
20 JULY 2007

Systematics of the First 2^+ Excitation with the Gogny Interaction

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PHYSICAL REVIEW LETTERS

week ending
19 JUNE 2009

First Gogny-Hartree-Fock-Bogoliubov Nuclear Mass Model

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The Gogny force: a successful mean field model

ELSEVIER

Nuclear Physics A 771 (2006) 103–168

Structure properties of even–even actinides at normal and super deformed shapes analysed using the Gogny force

J.-P. Delaroche ^a, M. Girod ^{a,*}, H. Goutte ^a, J. Libert ^b

PHYSICAL REVIEW C **84**, 054302 (2011)

Global systematics of octupole excitations in even-even nuclei

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PHYSICAL REVIEW C **91**, 044315 (2015)

Toward global beyond-mean-field calculations of nuclear masses and low-energy spectra

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The Gogny force: time odd fields

- Time odd fields come directly from the interaction. Some freedom in the density dependent part of the interaction

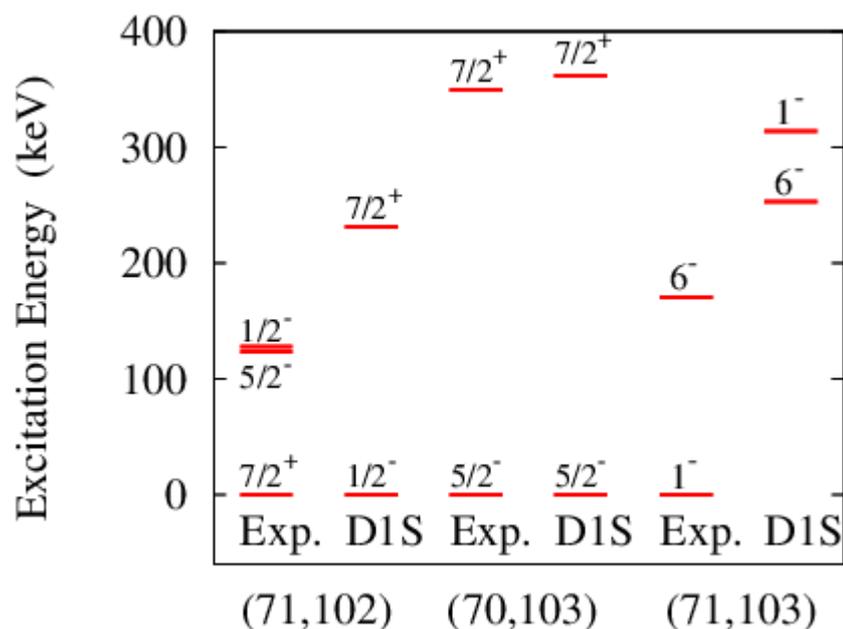
$$\Gamma_{\text{odd}} = \sum \bar{\nu} \rho_{\text{odd}}$$

Spin density dependence ?

- Time odd fields are not constrained by the fitting protocol
 - High spin physics
Direct comparison with experiment misleading
(dynamical pairing correlations, position of single particle energies, etc)
 - Odd mass nuclei
Direct comparison with experiment misleading
(dynamical pairing correlations, position of single particle energies, etc)
 - Odd-Odd nuclei
Gallagher-Moszkowski rule

Odd-odd systems and the Gallagher-Moszkowski rule

Gallagher-Moszkowski (GM) rule: In odd-odd systems with an unpaired proton K_p and neutron K_n a doublet is obtained with $J=K_n + K_p$ and $J=|K_n - K_p|$. The configuration with the lowest energy is that with parallel intrinsic spins.



- * Typical example: ¹⁷⁴Lu (Z=71)
- * Results for ¹⁷³Lu and ¹⁷³Yb also given
- * The inversion observed in ¹⁷³Lu explains why the 6⁻ - 1⁻ doublet is not the GS
- * GM rule is violated

GM rule is a consequence of the properties of the spin-spin neutron-proton nuclear interaction

Odd-odd systems and the Gallagher-Moszkowski rule

Analyzing the agreement of calculations with experimental data on the **ordering** of the doublets provides a handle on a poorly-determined part of the interaction

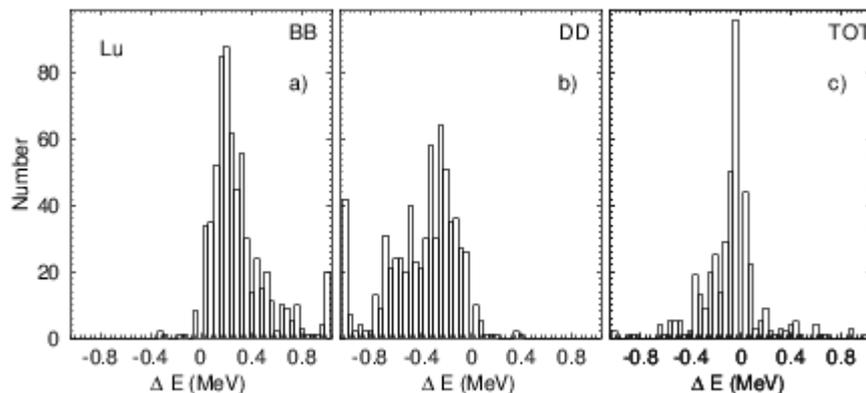
In Gogny two terms contribute to the splitting:

- Brink-Boecker central term

- Density dependent term $t_3(1 + x_0 P_\sigma)\delta(\vec{r}_1 - \vec{r}_2)\rho^\alpha(\vec{R})$

The region around Z=71 (Lu) is well known experimentally and the GM rule is fulfilled more than 95% of the cases

Perturbative calculation



Calculated GM doublet splittings
Lu isotopes (184-188)

Positive ΔE : agree with GM

BB: Brink-Boecker

DD: Density dependent

Odd-odd systems and the Gallagher-Moszkowski rule

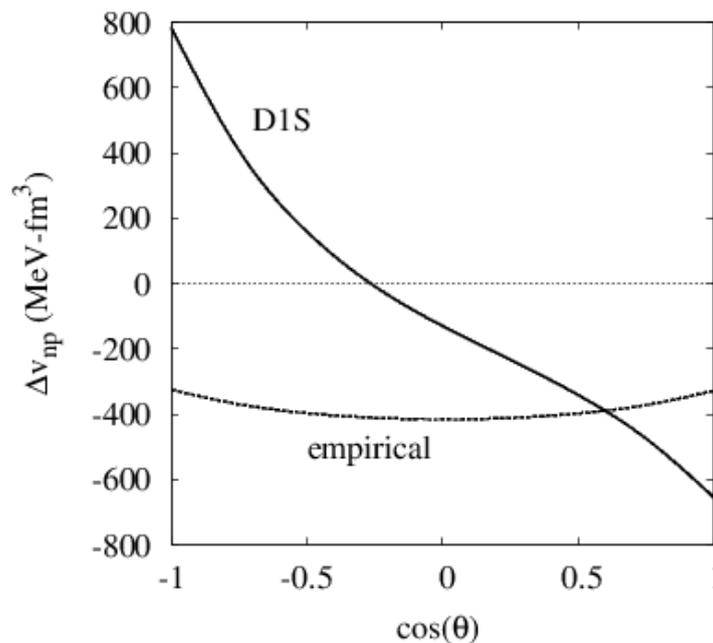
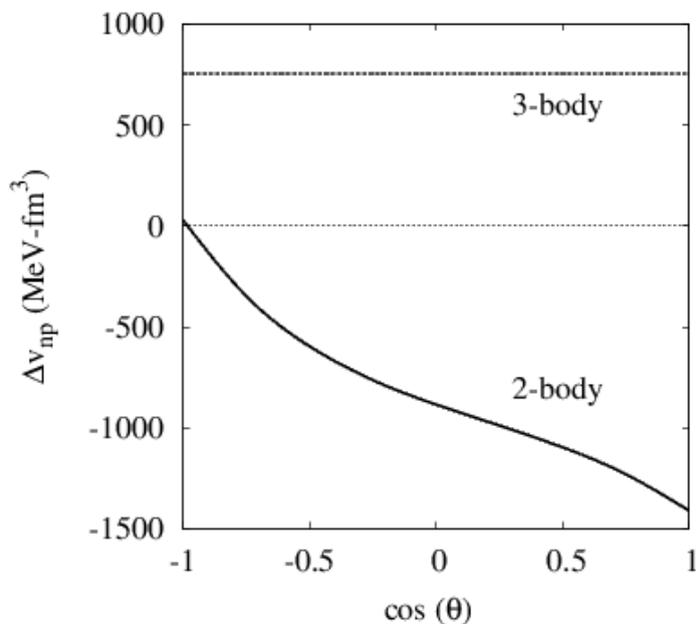
	2BP	3BP	FP	self-consistent
$^{164-168}\text{Ho}$	93%	8%	28%	45%
$^{168-172}\text{Tm}$	97	4	26	41
$^{172-176}\text{Lu}$	97	4	28	40
$^{180-184}\text{Ta}$	97	5	37	30
$^{184-188}\text{Lu}$	97	3	36	28

GM fails 60-70 % of the cases

BB contribution correct

DD contribution incorrect

The spin-spin neutron-proton density dependent interaction is wrong



Will a new density dependent term depending on spin-density solve it ?

The Gogny force: beyond the mean field

The Gogny force is an effective interaction where a density dependent term is added to simulate in-medium effects (saturation property)

$$V_{DD}(1, 2) = t_3 \delta(\vec{r}_1 - \vec{r}_2) (1 + x_0 P_\sigma) \rho^\alpha(\vec{R})$$

The density corresponds to the HFB state to be determined

$$E_{\text{HFB}} = \langle \Phi | H | \Phi \rangle \quad \rho(\vec{r}) = \langle \Phi | \hat{\rho}(\vec{r}) | \Phi \rangle$$

This non-linear dependence leads to “rearrangement terms” in the mean field

$$\partial \Gamma_{pq} = \sum_{rstu} \langle rs | \frac{\partial V_{DD}}{\partial \rho_{pq}} | tu \rangle \rho_{us} \rho_{tr}$$

And extra terms in the RPA residual interaction

$$\alpha = \frac{1}{3}$$

Non negligible effects as t_3 is of the order of thousand MeV

The Gogny force: beyond the mean field

In both Generator Coordinate Method and Symmetry restoration we need hamiltonian overlaps

$$\langle \Phi | \hat{H} | \Phi' \rangle / \langle \Phi | \Phi' \rangle \quad |\Phi\rangle \quad |\Phi'\rangle \quad \text{HFB wave functions}$$

Easy task using the generalized Wick theorem, provided the overlap is non-zero

Even in that case, the problem can be worked out as the required quantity is

$$\langle \Phi | \hat{H} | \Phi' \rangle = \left(\langle \Phi | \hat{H} | \Phi' \rangle / \langle \Phi | \Phi' \rangle \right) \langle \Phi | \Phi' \rangle$$

Is finite, provided the Pauli principle is enforced. Pole problem

Pauli principle (self-energy problem): Direct, exchange and pairing field have to be considered.

Real problem in many cases (specially when dealing with Coulomb)

The Gogny force: beyond the mean field

What about the density dependent part in overlaps ? $\langle \Phi | \hat{H} | \Phi' \rangle / \langle \Phi | \Phi' \rangle$

The obvious choice is the “**overlap density**” prescription

$$\rho(\vec{r})_{\text{ov}} = \langle \Phi | \hat{\rho}(\vec{r}) | \Phi' \rangle / \langle \Phi | \Phi' \rangle$$

But this is

- A **complex quantity**
- **Diverges for zero overlap**

Why not the “**correlated density**” (projected density)

$$|\Psi_\sigma\rangle = \int dq f_\sigma(q) |\Phi(q)\rangle$$

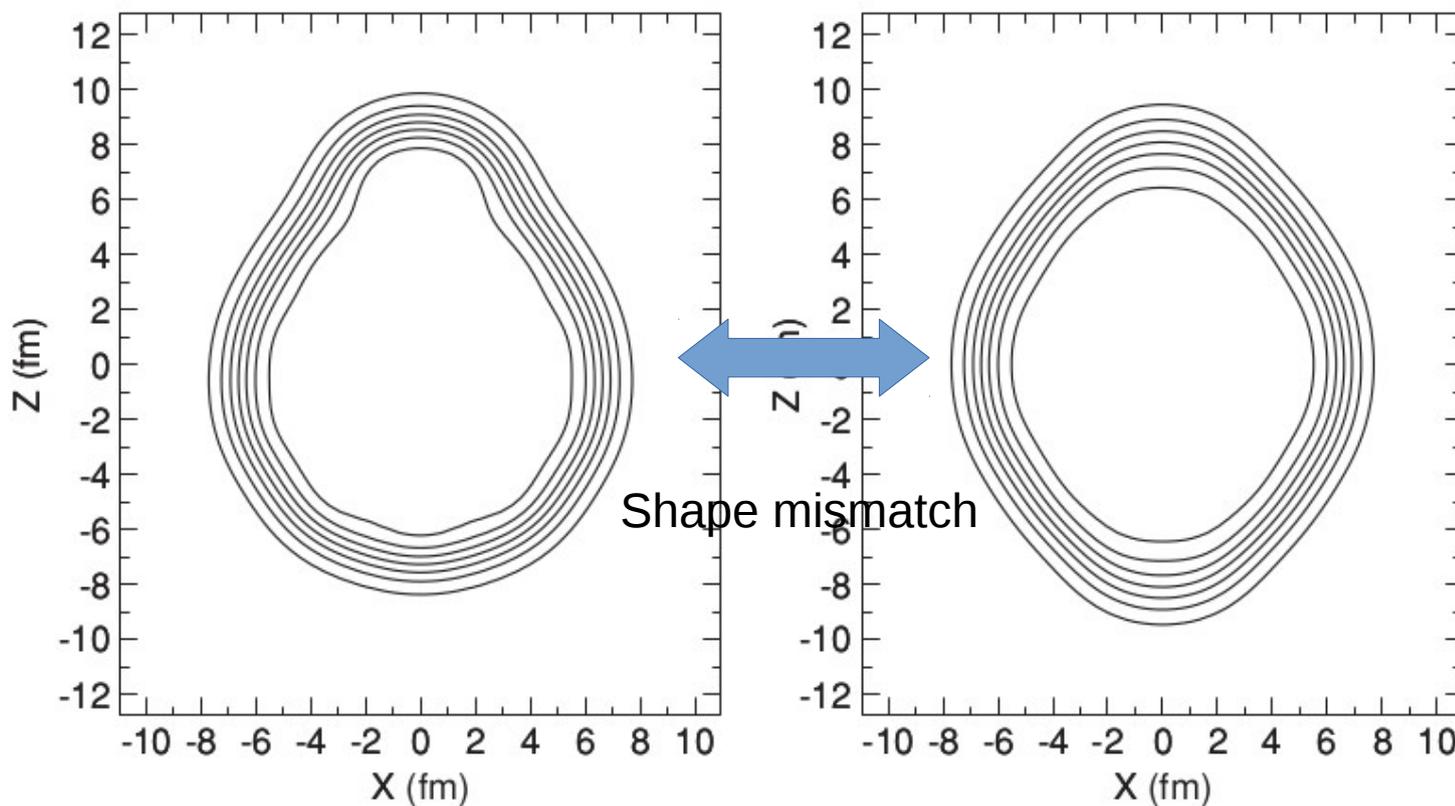
$$\rho(\vec{r})_{\text{corr}} = \langle \Psi_\sigma | \hat{\rho}(\vec{r}) | \Psi_\sigma \rangle / \langle \Psi_\sigma | \Psi_\sigma \rangle$$

Real, but **state dependent** !

The Gogny force: beyond the mean field

Let us consider the simplest case: parity projection

$$|\Psi_\pi\rangle = (1 + \pi\Pi)|\Phi(q)\rangle \quad \rho(\vec{r})_{\text{corr}} = \frac{\langle\Phi|\hat{\rho}(\vec{r})(1 + \pi\Pi)|\Phi\rangle}{\langle\Phi|(1 + \pi\Pi)|\Phi\rangle}$$



Intrinsic

Projected to positive parity

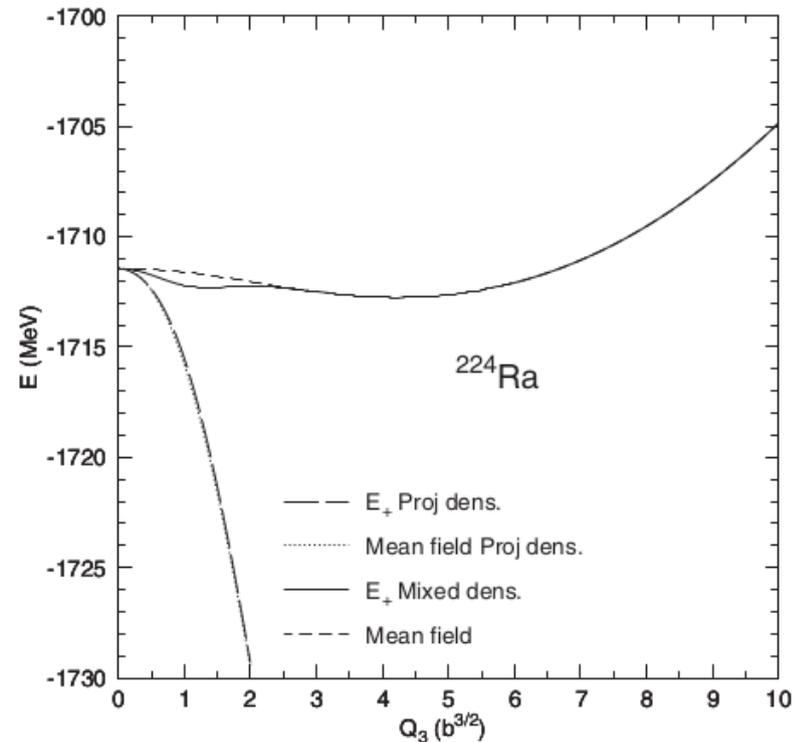
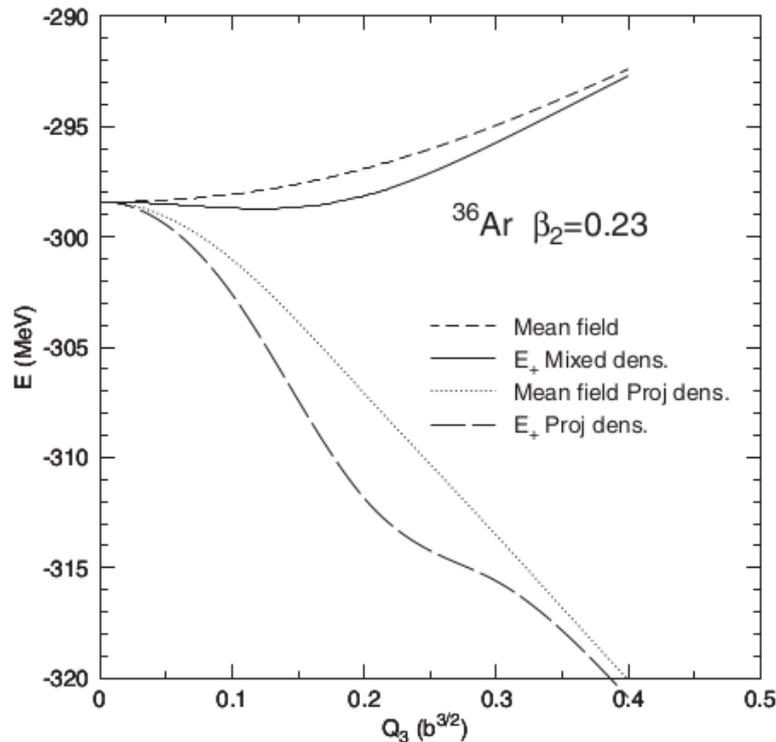
The Gogny force: beyond the mean field

$$\langle \Phi | H_{DD} | \Phi \rangle = \frac{t_3}{2} \sum_{\tau\tau'} \left(1 + \frac{x_0}{2} \right) - \left(x_0 + \frac{1}{2} \right) \delta_{\tau\tau'} \int d\vec{r}^3 \rho(\vec{r})_{\tau} \rho(\vec{r})_{\tau'} \rho(\vec{r})_{\text{corr}}^{\alpha}$$

The density mismatch leads to a reduction of the integral

J. Phys. G: Nucl. Part. Phys. **37** (2010) 064020

L M Robledo



With catastrophic consequences !!!! that rule out the correlated density

The Gogny force: beyond the mean field

The only alternative is the “overlap density”

- Real energies $\rho(\vec{r})_{\text{ov}}^* = \langle \Phi' | \hat{\rho}(\vec{r}) | \Phi \rangle / \langle \Phi' | \Phi \rangle$ $\rho(\vec{r})_{\text{ov}} = \langle \Phi' | \hat{\rho}(\vec{r}) | \Phi \rangle / \langle \Phi' | \Phi \rangle$
- Consistent with the rearrangement term in HFB entering the definition of the chemical potentials
- The GCM with the “overlap density” leads in the small amplitude limit to the RPA effective interaction of HFB

LMR Int. Jour. Of Mod Phys E16, 337

The only drawback is its (in general) complex value together with a non integer exponent α : Multivalued roots, Riemann sheets and “cuts” in the complex plane

Often the “overlap density” is “only negative” (due to symmetries) and $\alpha=1/3$ and you can find ways to circumvent the difficulties but Nature is not always forgiving (time reversal breaking HFB states, for instance)

The Gogny force: beyond the mean field

Obviously we have to find a better prescription for the density dependent part in the calculation of the overlaps encompassing also the evaluation of mp-mh overlaps

$$\langle \hat{H}_{\text{DD}} \beta_k^+ \beta_l^+ \beta_m^+ \beta_n^+ \rangle$$

One way is to approximate ρ^α by a polynomial in the density

Another is to replace the density dependent term by a genuine three body force (see D. Lacroix talk) but that makes calculations really painful (N^6 cost versus N^4 in evaluating matrix elements)

Also some attempts by using non-local three body forces (Gezerlis and Bertsch)

The problem is still open and fresh ideas are most welcome